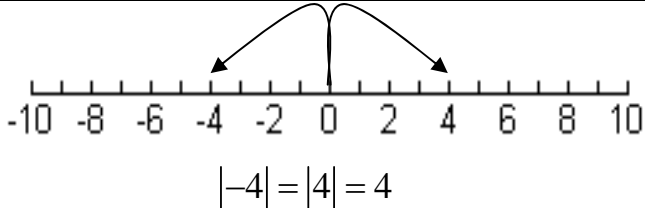
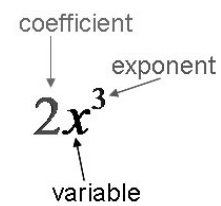
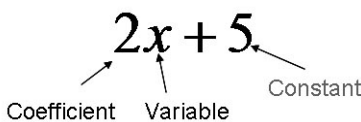
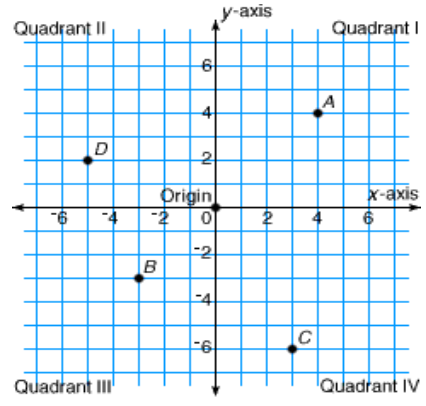
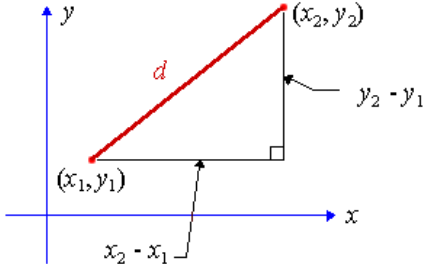
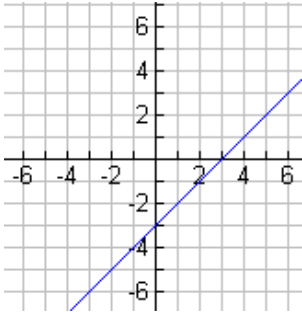
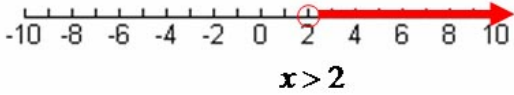
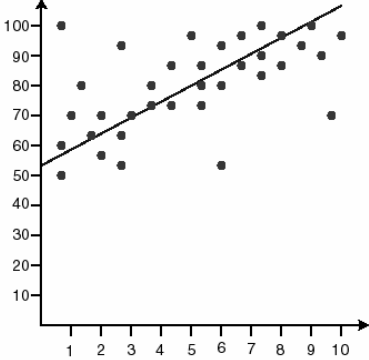
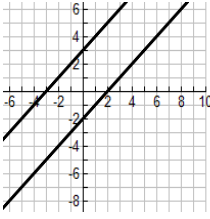
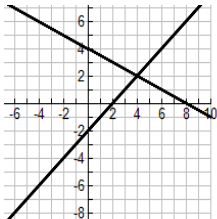
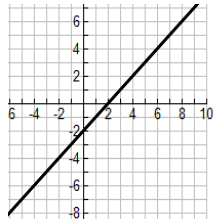
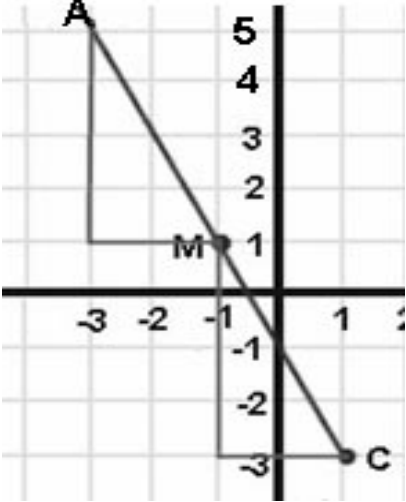
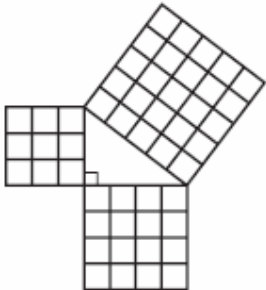


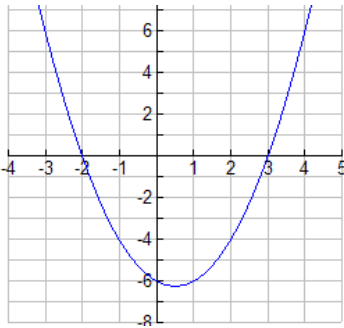
## Algebra I / Technical Algebra


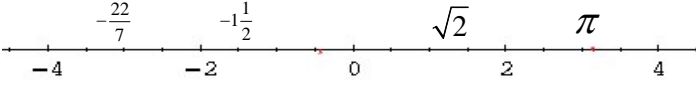
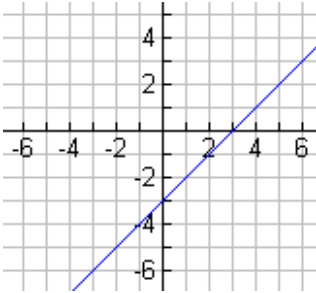
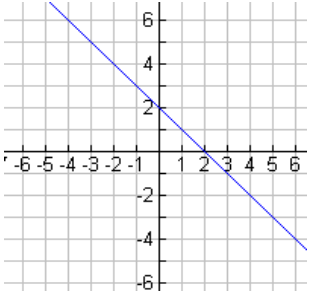
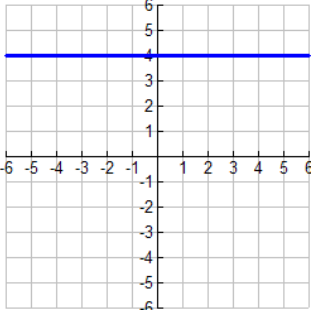
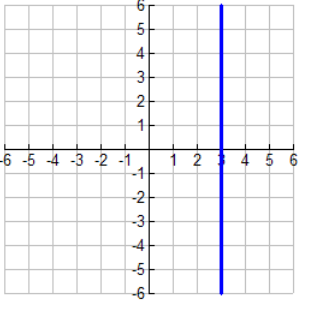
<p><b>Absolute Value:</b> A number's distance from zero on a number line. A number's absolute value is nonnegative.</p>	 <p>A number line from -10 to 10 with tick marks every 1 unit. Two arrows point from 0 to -4 and from 0 to 4. Below the line, the equation <math> -4  =  4  = 4</math> is written.</p>
<p><b>Algebraic Expressions:</b> A mathematical phrase that can include numbers, variables, and operation or grouping symbols. A combination of numbers and/or variables with at least one operation</p>	<p>Examples: Write an algebraic expression for the sum of six and a number: <math>6 + n</math> Evaluate the algebraic expression <math>-x^2 + 3</math> if <math>x = -4</math>: <math>-(-4)^2 + 3 = -(16) + 3 = -13</math></p>
<p><b>Coefficient:</b> The numerical factor of a variable term. A variable is multiplied by the coefficient in a term.</p>	 <p>The term <math>2x^3</math> is shown. An arrow points from the word 'coefficient' to the number 2. Another arrow points from the word 'exponent' to the number 3. A third arrow points from the word 'variable' to the letter x.</p>
<p><b>Combinations:</b> An arrangement of the elements of a set without regard to order. The number of combinations is given by</p> ${}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$ <p>(This can be read: given n elements, choose r, <math>n! = n \cdot (n-1)(n-2) \dots 1</math>, so <math>4! = 4 \cdot 3 \cdot 2 \cdot 1</math>)</p>	<p>The number of ways that 3 letters can be chosen from the letters A,B,C,D,E is ten:  <math>\{A, B, C\}</math> <math>\{A, B, E\}</math> <math>\{A, C, E\}</math> <math>\{B, C, D\}</math>  <math>\{B, D, E\}</math> <math>\{A, B, D\}</math> <math>\{A, C, D\}</math> <math>\{A, D, E\}</math>  <math>\{B, C, E\}</math> <math>\{C, D, E\}</math></p> $\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot 3!}{3!(2)} = \frac{20}{2} = 10$
<p><b>Constants:</b> A term that has no variable factor.</p>	 <p>The expression <math>2x + 5</math> is shown. An arrow points from the word 'Coefficient' to the number 2. Another arrow points from the word 'Variable' to the letter x. A third arrow points from the word 'Constant' to the number 5.</p>
<p><b>Coordinate Plane:</b> A plane formed by a horizontal number line (x-axis) and a vertical number line (y-axis). Points on the coordinate plane are defined by their horizontal and vertical distance from the origin (0,0).</p>	 <p>A coordinate plane with x and y axes ranging from -6 to 6. The origin (0,0) is labeled. Four points are plotted: A at (4, 4), B at (-2, -3), C at (4, -6), and D at (-4, 2). The quadrants are labeled: Quadrant I (top-right), Quadrant II (top-left), Quadrant III (bottom-left), and Quadrant IV (bottom-right).</p>

<p><b>Distance Formula:</b> The distance <math>d</math> between any two points <math>(x_1, y_1)</math> and <math>(x_2, y_2)</math> is</p> $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$								
<p><b>Domain &amp; Range:</b>  <b>Domain:</b> The set of all x-coordinates in the ordered pairs <math>(x,y)</math> of a relation.  <b>Range:</b> The set of all the y-coordinates in the ordered pairs <math>(x,y)</math> of a relation.  The domain of a function is the set of all <math>x</math> values for which the function is defined.</p>	<table border="1" data-bbox="963 569 1070 758"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> </tr> <tr> <td>0</td> <td>0</td> </tr> <tr> <td>-1</td> <td>1</td> </tr> </tbody> </table> <p>Domain: <math>\{1,0,-1\}</math>      Range: <math>\{1,0\}</math></p>	x	y	1	1	0	0	-1	1
x	y								
1	1								
0	0								
-1	1								
<p><b>Equations (solving, graphing, slope-intercept, etc.):</b> An equation is a mathematical sentence containing an equal sign.  To solve an equation, find values for the variable that makes the sentence true.  The graph of a linear equation contains all points that make that equation true.</p>	<p><math>y = x - 3</math>  <math>m = 1</math>  y-intercept = <math>-3</math></p> 								
<p><b>Factoring:</b> To write a number (or expression) as a product of two or more numbers (or expressions).</p>	<p>Some common factoring patterns are:</p> <p>Common factor : <math>x^2y + 2x^3y = x^2y(1 + 2x)</math>  Difference of squares : <math>x^2 - y^2 = (x - y)(x + y)</math>  Grouping : <math>x(a + b) - y(a + b) = (a + b)(x - y)</math></p>								
<p><b>Function Notation:</b> Use the symbol <math>f(x)</math> to mean the function whose input is <math>x</math>. “<math>f(x)</math>” is read “<math>f</math> of <math>x</math>” and means that the value of the function depends on the value of <math>x</math>. <math>f(x)</math> is the output (dependent variable) of the function with input (independent variable) <math>x</math>.</p>	<p>A function is a mapping from a domain to a range. The graph of a function will pass the <i>vertical line test</i>, that is, any vertical line drawn on the graph will only cross the graph once.  If <math>f(x) = x + 3</math> then <math>f(0) = 3</math>, <math>f(1) = 3</math> and <math>f(-1) = 2</math></p>								
<p><b>Inequality:</b> The graph of a mathematical sentence showing the relationship between quantities that are not equal, using <math>&lt;</math>, <math>&gt;</math>, <math>\leq</math>, <math>\geq</math>, or <math>\neq</math>.</p>	 <p><math>x &gt; 2</math></p>								

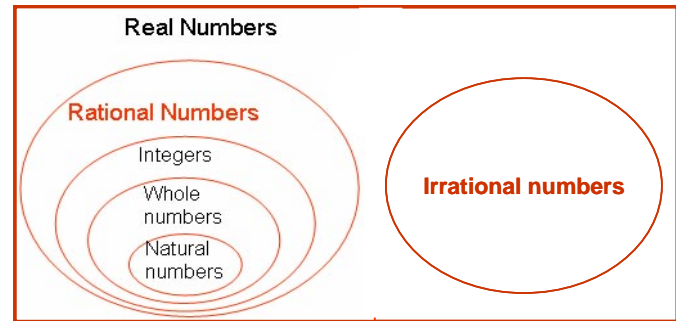
<p><b>Inverse Operations (algebraic):</b> Operations that undo each other: <math>b</math> and <math>-b</math> are additive inverses. <math>b</math> and <math>\frac{1}{b}</math> are multiplicative inverses (<math>b \neq 0</math>).</p>	<p>Addition and subtraction are inverse operations (undo adding 3 by subtracting 3)</p> <p>Multiplication and division are inverse operations (undo multiplying by 2 by dividing by 2)</p>
<p><b>Irrational Numbers:</b> A number that cannot be written as a ratio of two integers. Irrational numbers in decimal form are non-terminating and non-repeating.</p>	<p>These numbers are irrational:</p> $\sqrt{2} = 1.414213562... \quad 0.01011011101111...$ $\pi \doteq 3.14159265358979323846264338327950...$ <p>These are rational:</p> $\sqrt{4} = 2 \quad 0.01010101... = \frac{1}{99}$
<p><b>Line of Best Fit:</b> A straight line that best fits the data on a scatter plot. (This line may pass through some, none, or all of the points). The line of best fit is a trend line for the data.</p>	<p>The line of best fit for this data suggests a correlation between the x and y values.</p> 
<p><b>Linear Systems:</b> A linear system of equations is a set of two equations with the same variables, graphed in the same coordinate plane. If the system has a solution, it is called consistent. If it does not have a solution it is inconsistent.</p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p><math>x - y = -3</math> <math>x - y = 2</math></p> <p>No solution: Parallel lines</p> </div> <div style="text-align: center;">  <p><math>x + 2y = 8</math> <math>x - y = 2</math></p> <p>One solution: (4,2)</p> </div> <div style="text-align: center;">  <p><math>x - y = 2</math> <math>2x - 2y = 4</math></p> <p>Infinite number of solutions: same line</p> </div> </div> <p>Linear systems can be solved graphically, by substitution or by elimination (linear combination).</p>

<p><b>Midpoint Formula:</b> The midpoint, M, of a line segment with endpoints <math>(x_1, y_1)</math> and <math>(x_2, y_2)</math> is</p> $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	<p>The point M: (-1,1) is the midpoint of the segment AC with endpoints A: (-3,5) and C: (1,-3).</p> 
<p><b>Permutations:</b> An arrangement of elements of a set in which order is important. The number of permutations is given by <math>{}_n P_r = \frac{n!}{(n-r)!}</math></p>	<p>How many ways can two letters be arranged from the four letters M, A, T, and H?</p> <p>There are 12 possible permutations: MA, AM, MT, TM, MH, HM, AT, TA, AH, HA, TH, HT</p> ${}_4 P_2 = \frac{4!}{(4-2)!} = \frac{4 \cdot 3 \cdot 2}{(2)} = 12$
<p><b>Polynomial:</b> A monomial or the sum/difference of two or more monomials. A quotient with a variable in the denominator is not a polynomial.</p>	<p>These are polynomials:</p> <p>Monomial: <math>5x</math>                      Binomial: <math>2x + 6</math> Trinomial: <math>3x^2 + 2x - 1</math>      Polynomial: <math>x^3 - 3x^2 + 2x - 1</math></p> <p>Theses are not polynomials: <math>3x^{-1} + 2x - 4</math>, <math>\frac{5}{x^2} - x</math>, <math>x + \sqrt{x} + 2</math></p>
<p><b>Pythagorean Theorem:</b> The Pythagorean Theorem states that, in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the two legs. There are several proofs that use area models.</p>	<p>Pythagorean Theorem: <math>a^2 + b^2 = c^2</math> where <math>a</math> and <math>b</math> are the lengths of the legs of the right triangle and <math>c</math> is the length of the hypotenuse.</p> <p>This is an area model to demonstrate the Pythagorean Theorem.</p> 

<p><b>Quadratic Equation:</b> A function given by a polynomial equation of degree two. Its graph is a parabola.</p>	<p>A quadratic equation can be expressed in the form <math>ax^2 + bx + c = 0</math>. The quadratic term is <math>ax^2</math>, the linear term is <math>bx</math>, and the constant term is <math>c</math>. The real roots (solutions) to the equation are the x-intercepts of its graph. The solutions to <math>x^2 - x - 6 = 0</math> are the x-intercepts of the graph <math>y = x^2 - x - 6</math>: <math>x = -2</math> or <math>x = 3</math></p> 
<p><b>Quadratic Formula (discriminant):</b> The quadratic formula is used to find the roots/solutions to quadratic equations of the form <math>ax^2 + bx + c = 0</math>. Solving the quadratic equation for <math>x</math> by completing the square yields the quadratic formula which states that <math>x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}</math></p> <p>The discriminant is the part of the quadratic formula that is under the radical. It determines the nature of the roots (what kind and how many).</p>	<p>What is the nature of the roots of the following quadratic equations? What kind of roots are there and how many?</p> <ol style="list-style-type: none"> <li><math>x^2 - 2x - 3 = 0</math>, <math>D = b^2 - 4ac = 4 - (-12) = 16</math> <math>D</math> is a positive perfect square, so two real rational roots. (<math>x = 3</math> or <math>x = -1</math>)</li> <li><math>x^2 - 3x - 3 = 0</math>, <math>D = b^2 - 4ac = 9 - (-12) = 21</math> <math>D</math> is positive, so two real irrational roots. <math>\left(x = \frac{3}{2} \pm \frac{\sqrt{21}}{2}\right)</math></li> <li><math>x^2 - 3x + 3 = 0</math>, <math>D = b^2 - 4ac = 9 - (12) = -3</math> <math>D</math> is negative, so two complex conjugate roots. <math>\left(x = \frac{3}{2} \pm \frac{\sqrt{3}}{2}i\right)</math></li> <li><math>x^2 - 6x + 9 = 0</math>, <math>D = b^2 - 4ac = 36 - (36) = 0</math> <math>D</math> is zero, so one real double root. (<math>x = 3</math>)</li> </ol>

<p><b>Ratio/Proportion (scale factors):</b> Ratio: A comparison of two numbers by division.  <b>Proportion:</b> An equation that states that two ratios are equal.</p> <p><b>Scale factor:</b> The ratio by which a drawing or figure is enlarged or reduced. The resultant figure is similar to the original.</p>	<p>If the Eiffel Tower is 1000 feet tall and the drawing of it is 10 inches tall, the scale would be:  10 inches=1000 feet or 1 inch = 100 feet.</p> <p>A scale factor of one hundred means that the linear dimensions of the tower are 100 times that of the drawing. However, the area of the tower is <math>(100)^2</math> or 10,000 times that of the figure in the drawing.</p> 
<p><b>Real number:</b> A number that is either rational or irrational. Real numbers can be represented by the set of infinite points on a number line.</p>	<p>Real numbers include all of the following numbers: rational, irrational, and thus integers, whole numbers, natural numbers, zero.</p> <p>The properties of real numbers include the commutative, associative, distributive, additive and multiplicative identity, and additive and multiplicative inverse properties.</p> 
<p><b>Slope:</b> For a linear equation, the ratio of the vertical change to the horizontal change between two points on the graph of its line. It measures the steepness of the line.</p>	<div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  <p>Positive slope</p> </div> <div style="text-align: center;">  <p>Negative slope</p> </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div style="text-align: center;">  <p>Zero slope</p> </div> <div style="text-align: center;">  <p>No or undefined slope</p> </div> </div>

**Subsets:**  $A$  is a subset of  $B$  if every member of  $A$  is also a member of  $B$ . Every set is a subset of itself. The empty set is a subset of every set. The number of subsets of a given set is  $2^n$  where  $n$  is the number of elements in the set.



Natural numbers is a subset of whole numbers which is a subset of integers which is a subset of rational numbers which is a subset of real numbers.

Irrational numbers is also a subset of real numbers.